Geometric probability and circle review

Lesson objectives: to learn about probability from a geometric perspective and how to apply that knowledge to solve problems. Also, to review the properties of circles such as circumference, arc length and area and how to use them to solve problems.

Section 10-8
Geometric Probability

What is geometric probability? It is geometric ratio (area to area, or length to length, for example) of an event happening versus it not happening.

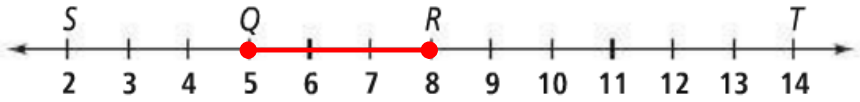
Probability and Length

Point S on $\overline{AD}$ is chosen at random. The probability that S is on $\overline{BC}$ is the ratio of the length of $\overline{BC}$ to the length of $\overline{AD}$.

$$P(S \text{ on } \overline{BC}) = \frac{BC}{AD}$$

So if AD has length 12 and BC has length 4, then the probability that point S is on BC is 4/12 or 1/3.

Example



Point K on $\overline{ST}$ is chosen at random. What is the probability that K lies on $\overline{QR}$?

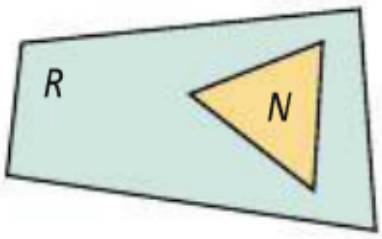
$$P(K \text{ on } \overline{QR}) = \frac{\text{length of } \overline{QR}}{\text{length of } \overline{ST}} = \frac{|5-8|}{|2-14|} = \frac{3}{12}, \text{ or } \frac{1}{4}$$

The probability that K is on $\overline{QR}$ is $\frac{1}{4}$, or 25%.

Probability and Area

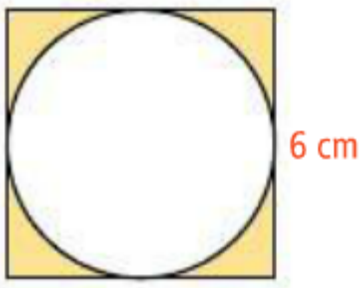
Point S in region R is chosen at random. The probability that S is in region N is the ratio of the area of region N to the area of region R.

$$P(S \text{ in region } N) = \frac{\text{area of region } N}{\text{area of region } R}$$



Example

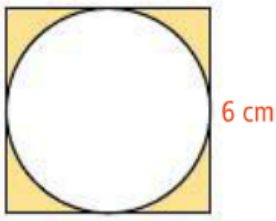
A circle is inscribed in a square. Point Q in the square is chosen at random. What is the probability that Q lies in the shaded region?



Example (continued)

$$\text{area of shaded region} = \text{area of square} - \text{area of circle}$$
$$= 6^2 - \pi(3)^2$$
$$= 36 - 9\pi$$

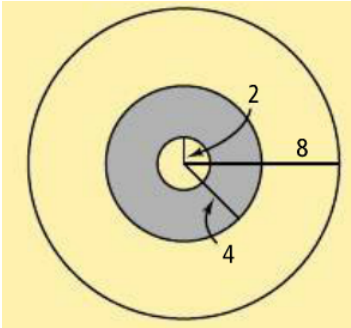
$$P(Q \text{ lies in shaded region}) = \frac{\text{area of shaded region}}{\text{area of square}}$$
$$= \frac{36 - 9\pi}{36} \approx 0.215$$



The probability that Q lies in the shaded region is about 0.215, or 21.5%.

Example

Suppose a dart lands randomly on the target below. What is the probability that the dart will land in the shaded region?



$$P(A_{shaded}) = \frac{A_{middle} - A_{inner}}{A_{outer}} = \frac{\pi(4)^2 - \pi(2)^2}{\pi(8)^2} = 18.75\%$$

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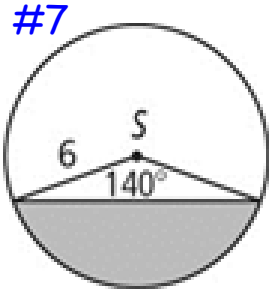

Area of $\Delta = \frac{1}{2}(\text{side}_1)(\text{side}_2)(\sin(140°))$

Area of sector $= \frac{140°}{360°} * \pi (6)^2$

Area of a segment
= area of the sector minus the area of the triangle.
= 32.4 sq. units

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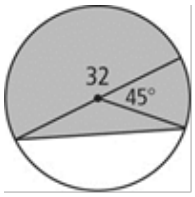

Area shaded =

Area circle - Area segment

Area shaded
$= 256\pi - (96\pi - 64\sqrt{2})$
$= 160\pi + 64\sqrt{2}$

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Area segment = area sector − area Δ

1. Area sector $= \frac{135°}{360°} * \pi (16)^2$
$= 96\pi$

2. Area $\Delta = \frac{1}{2}(16)(16)(\sin(135))$
$= 128 * \frac{\sqrt{2}}{2}$
$= 64\sqrt{2}$ (or 90.51)

3. Area segment $= 96\pi - 64\sqrt{2}$

4. Area circle $= \pi r^2$
$= \pi (16)^2$
$= 256\pi$